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Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

$$\int_0^x y dx = y - x. \quad \therefore y dx = dy - dx, \text{ or } dx = \frac{dy}{y+1}.$$

$x = \log(y+1)$, and $e^x = (y+1)$ is the required equation.

Also solved by S. A. COREY, F. P. MATZ, and the Proposer.

DIOPHANTINE ANALYSIS.

127. Proposed by F. P. MATZ, Sc. D., Ph. D., Reading, Pa.

Can there be determined three cube numbers whose sum is the product of two squares?

I. Solution by A. H. HOLMES, Brunswick, Maine.

Take x^3 , x^3y^3 , and x^3z^3 , for the three cube numbers, and u^2 and v^2 for the square numbers.

$\therefore x^3(1+y^3+z^3) = u^2v^2$. If we put $y=2$, $z=3$, $1+2^3+3^3=6^2=u^2$, $x^3=n^2=w^6$. $\therefore x=w^2$, $v=w^3$. Put $w=2$. $\therefore x=4$, $v=8$.

$\therefore 64+512+1728=36 \times 64=2304$, or $4^3+8^3+12^3=6^2 \times 8^2$.

If it is required that no two of the numbers to be cubed and squared should be the same, put $x^3=uv$, so that $1+y^3+z^3=uv$.

Put $x=9$, $y=6$, and $z=8$. Then $u=3$, $v=243$.

$\therefore 9^3+54^3+72^3=3^2 \times 243^2$.

II. Solution by J. EDWARD SANDERS, Hackney, Ohio.

If $a^3+b^3+c^3=d^3=(e.f)^3$, then one set of numbers satisfying the condition is: $(a.d^{2n-1})^3+(b.d^{2n-1})^3+(c.d^{2n-1})^3=d^{6n}=(e^{3n})^2.(f^{3n})^2$.

Again, since $1^3+2^3+3^3=2^2.3^2$, another solution is $(a^4)^3+(2a^4)^3+(3a^4)^3=(2a^3)^2.(3a^3)^2$.

Also solved by G. B. M. ZERR, R. D. Carmichael, and the Proposer.

GEOMETRY.

NOTE. Problems 259 and 261 are identical. A solution may be found in Lachlan's *Modern Pure Geometry*, pp. 241-2.

254. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

Find the cartesian equation of a line that is both tangent and normal to the cardioid.

Solution by F. H. SAFFORD, Ph. D., The University of Pennsylvania.

Let the required line be $2ax+2by+c=0$, which is to be both tangent and normal to $(x^2+y^2+cx)^2=c^2(x^2+y^2)$.

The constants a and b are to be found so as to satisfy the given conditions.

Transform the line and cardioid through the inversion

$$x = \frac{cx'}{x'^2 + y'^2}, \quad y = \frac{cy'}{x'^2 + y'^2}$$

obtaining, after dropping accents,

$$x^2 + y^2 + 2ax + 2by = 0; \quad y^2 = 2x + 1.$$

Since angles are not changed by inversion the conditions to be satisfied are now that the circle shall be both tangent and normal to the parabola.

The latter condition requires that the coördinates of one intersection shall satisfy $y^2 + by - x - a = 0$.

Elimination of y^2 from these three equations followed by elimination of y from the two equations thus obtained gives $(x+1)(x+2a-1)=0$.

The first root is not used as it leads to an imaginary intersection.

From the second root the intersection is

$$[1-2a, \pm \sqrt{(3-4a)}], \text{ also } b = \frac{3a-2}{\pm \sqrt{(3-4a)}}.$$

With this value of b the abscissae of intersections of circle and parabola are given by

$$(x^2 + 2ax + 2x + 1)^2 - \frac{4(3a-2)}{3-4a}(2x+1) = 0.$$

Removing the factor $x+2a+1$ which corresponds to the orthogonal intersection, the result is the following cubic,

$$x^3 + x^2(2a+5) + 11x + \frac{13-18a}{3-4a} = 0.$$

As the circle is to be tangent to the parabola this equation must have a double root, *i. e.*, its discriminant must vanish, thus giving the following equation for the determination of a ,

$$9a^5 + 24a^4 - 143a^3 + 202a^2 - 116a + 24 = 0.$$

The roots of the latter are 1, 1, $\frac{2}{3}$, $\frac{2}{3}$, -6 , of which the first four lead to imaginary intersections.

When a is -6 , b is $\frac{\mp 20\sqrt{3}}{9}$, so that the desired result is*

$$108x \pm 40\sqrt{3}y - 9c = 0.$$

*Dr. Hoover obtains the obvious line $y=0$, but not the ones of the published solution. Dr. Safford's fortunate choice of c for absolute term in the assumed line excludes the line $y=0$ from his result. A perfectly general method would furnish a singly infinite family of lines only the above three of which are real. An investigation of this family for, say the general nodal-circular quartic, ought to lead to some interesting results. Ed. G.